Int. J. Heat Mass Transfer. Vol. 34, No. 7, pp. 1895–1897, 1991
Printed in Great Britain

0017–9310/91 $3.00+0.00

TECHNICAL NOTES

Apparent absorptance for diffusely and specularly reflecting spherical cavities

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(*Received* 29 *November* 1989 *and in final form* 10 *July* 1990)

INTRODUCTION

THE APPARENT absorptance of a cavity, α_{cavity}, is defined as the fraction of energy flux emitted by a black-body surface stretched across the cavity opening that is absorbed by the cavity walls.‡ Because of multiple reflections among the cavity walls, α_{cavity} will exceed the surface absorptance, α. Such an effect, called the cavity effect, is employed in the design of receiver/reactors for solar furnaces to capture the solar radiation. The cavity effect for partial enclosures has been analyzed for specific configurations, among them: cylindrical, conical, spherical, rectangular-groove and V-groove cavities [1–3]. These problems have been solved for specularly and diffusely reflecting cavity walls. Little information is presently available which describes the particular case of specularly reflecting spherical cavities with a circular opening. Such cavities offer several intriguing advantages over the conventional heavily insulated enclosures made of ceramic materials when applied for solar furnace receivers at moderate to high (2000 K and above) temperatures [4]. Specular spherical geometries have also significant importance in modeling radiant heat transport through the void spaces across a porous material, and the apparent surface emittance was analytically derived for a differential element of a sphere [5, 6]. In the present study, the apparent absorptance of a spherical cavity with a circular opening is calculated for cavity walls having both specular and diffuse reflectance components.

ANALYSIS

The space inside the cavity is assumed to be a non-participating medium. The walls of the cavity are taken to be isothermal at 0 K, opaque ($\alpha = 1-\rho$), gray, and their reflectivity is subdivided into diffuse and specular components, ρ^d and ρ^s, respectively [7] ($\rho = \rho^d + \rho^s$). Radiation entering the cavity opening from an external source is assumed to be uniformly and diffusely distributed. For the case of pure diffusely reflecting walls ($\rho^s/\rho = 0$), the apparent cavity absorptance, α_{cavity}, is given by [3]

$$\alpha_{cavity} = \frac{\alpha}{1-\frac{1}{2}(1-\alpha)(1+\cos\Phi)} \tag{1}$$

where α is the surface absorptance and Φ the opening cone angle defined in Fig. 1.

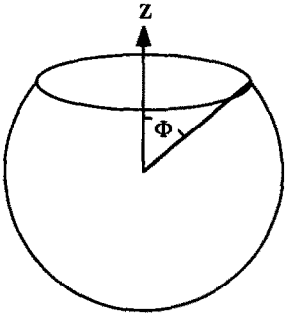

FIG. 1. Spherical cavity with a circular opening.

For cavities showing some specularity a Monte-Carlo ray-trace simulation was used. This method has been widely used for the analysis of radiative transport [8]. It consists of following probable paths of a large number of discrete bundles of energy being diffusely and uniformly emitted from the circular opening. With Cartesian coordinates centered at the center of the spherical cavity and its x-axis normal to the circular opening, as shown in Fig. 1, the equation of the sphere of unit radius is

$$F(x,y,z) = x^2+y^2+z^2-1 = 0. \tag{2}$$

Our incident ray, or energy bundle path, enters the cavity through the opening at point $P_1(x_1,y_1,\cos\Phi)$ and has a direction parallel to the unit vector $\mathbf{u} = u_1\mathbf{j}+u_2\mathbf{j}+u_3\mathbf{k}$. The equation that gives the coordinates of a generic point P_i on the ray is, in vectorial notation

$$(P_i - P_1)\times\mathbf{u} = 0 \tag{3}$$

with

$$|\mathbf{u}| = 1. \tag{4}$$

Since the circular opening emits in a diffuse and uniform manner, the number of energy bundles emitted from a certain ring of radius r on the opening are proportional to its area $2\pi r\,dr$, and their direction is chosen randomly from a set that is weighted according to a cosine distribution. Thus

$$\mathbf{u}\cdot\mathbf{k} = \cos(\sin^{-1}\sqrt{\mathscr{R}_1}) = \sqrt{(1-\mathscr{R}_1)} \tag{5}$$

and

$$\sqrt{(x_1^2+y_1^2)} = \sin\Phi\sqrt{\mathscr{R}_2} \tag{6}$$

where $\mathscr{R}_1$ and $\mathscr{R}_2$ are random numbers drawn from a uniformly distributed set between 0 and 1. Once we know the direction of an individual bundle, we can determine the point of incidence on the spherical cavity. Let $P_2(x_2,y_2,z_2)$ be the

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‡ For gray-walled cavities, the apparent absorptance is equivalent to the apparent emittance, defined as the ratio of the energy flux emitted by the cavity walls streaming out of the cavity opening to that streaming out of a black-walled cavity [1, 2].

point of intersection of the ray with the cavity walls. Its coordinates can be obtained by solving the system of equations (2) and (3). Here, a random choice, depending on the surface absorptance, determines whether the bundle is absorbed or reflected. If absorbed, a count is recorded and its history is terminated. If reflected, another random choice, depending on ρ^s/ρ, will determine whether the reflection is specular or diffuse. The direction of the reflected ray is expressed accordingly by the vector equation:

for specular reflection [9]

$$\mathbf{r} = \mathbf{u} - 2(\mathbf{n}\cdot\mathbf{u})\cdot\mathbf{n}; \tag{7}$$

for diffuse reflection

$$\mathbf{r}\cdot\mathbf{n} = \sqrt{(1-\mathscr{R}_3)} \tag{8}$$

where $\mathbf{u}$ and $\mathbf{r}$ are unit vectors along the incident and reflected rays, $\mathbf{n}$ the unit vector along the normal at P_2 pointing into the reflecting surface, and $\mathscr{R}_3$ is another random number between 0 and 1. The equation that gives the coordinates of a generic point P_r on the reflected ray is, in vectorial notation

$$(P_r - P_2)\times\mathbf{r} = 0 \tag{9}$$

with

$$|\mathbf{r}| = 1. \tag{10}$$

Again, we can determine the new point of incidence on the cavity and repeat the algorithm to follow the path of the energy bundle until absorption takes place. Eventually, each bundle of energy will have either been absorbed by the cavity surface after multiple reflections or have left through the opening. This procedure is repeated for a large enough sample of energy bundles so that the results are statistically meaningful.

RESULTS

Using a sample of 100 000 rays, we have counted the number of absorptions by the cavity walls and divided by the total number of bundles of energy emitted. The accuracy of the numerical computation was checked against the exact analytical solutions available for the specularly reflecting conical cavity and the diffusely reflecting spherical cavity [2, 3]. The results, believed to be accurate within 5%, are presented in Fig. 2 as a function of the opening angle Φ for various values of surface absorptance α. The data points were calculated by the Monte-Carlo method for the pure specular reflecting walls ($\rho^s/\rho = 1$). The lines drawn in association with them are the results of the exact analytical solution for the pure diffuse case ($\rho^s/\rho = 0$).

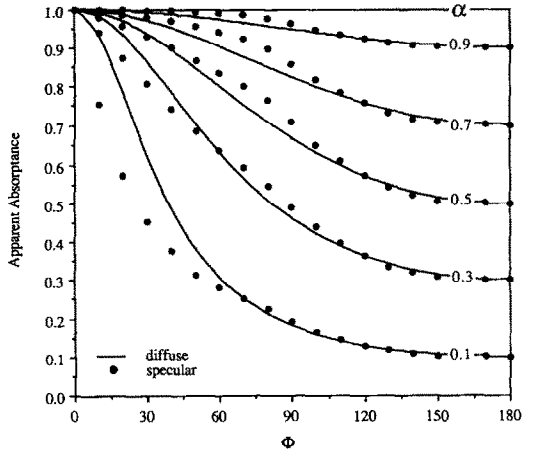

FIG. 2. Apparent absorptance as a function of the cone-opening angle for various values of the surface absorptance.

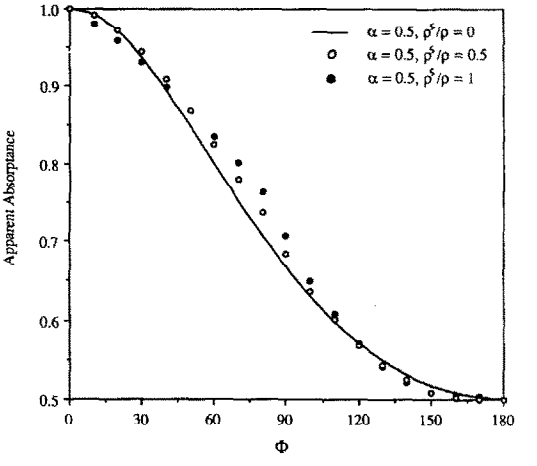

FIG. 3. Apparent absorptance as a function of the cone-opening angle for a surface absorptance of 0.5 and $\rho^s/\rho = 0$, 0.5, 1.

As expected, the apparent absorptance, α_{cavity}, approaches unity as the opening angle becomes smaller, and decreases monotonically approaching the surface absorptance as the opening angle becomes bigger. However, in contrast to the results for conical, cylindrical, rectangular-groove and V-groove cavities, the specularly reflecting spherical cavity does not always give higher values of α_{cavity} than does a diffusely reflecting one. For $\alpha = 0.1$, the contrary is true for every opening angle, and the difference can be as high as 25% for small opening angles. For $\alpha = 0.3$, 0.5, 0.7, and 0.9, the values of α_{cavity} for specular reflectance alternates the curve for the diffuse case, giving generally higher values when $60° < \Phi < 90°$. Similar to the results for cylindrical cavities [7], the curves for $\rho^s/\rho = 0.5$ (shown in Figs. 3 and 4 for surface absorptances of 0.5 and 0.1, respectively) lie halfway between those for purely specular and purely diffuse reflectance when the surface absorptance is high. At lower values of surface absorptance, the curves for $\rho^s/\rho = 0.5$ lie closer to those for diffuse reflectance.

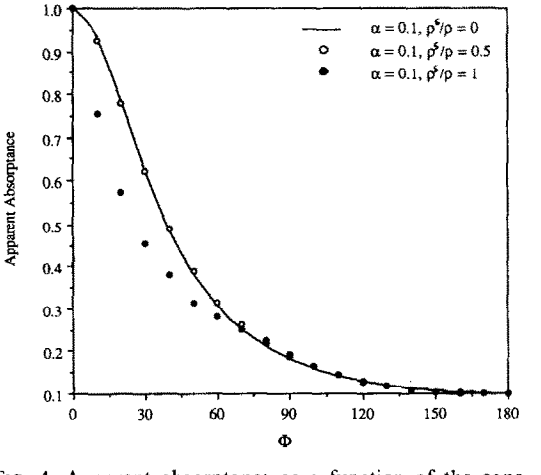

FIG. 4. Apparent absorptance as a function of the cone-opening angle for a surface absorptance of 0.1 and $\rho^s/\rho = 0$, 0.5, 1.

Acknowledgements—The author is very grateful to The Northern States Power Co. for the financial support, and to Prof. E. A. Fletcher and Prof. E. M. Sparrow who made several helpful suggestions.

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Int. J. Heat Mass Transfer. Vol. 34, No. 7, pp. 1897–1901, 1991
Printed in Great Britain

0017–9310/91 $3.00 + 0.00

The extension of Boadway's transformation technique to two or more dimensional moving boundary problems

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(*Received* 16 *February* 1990 *and in final form* 19 *July* 1990)

1. INTRODUCTION

NUMEROUS technical problems involve the movement of a phase boundary induced by the diffusion of energy or mass. Most common examples including the conduction of heat are the solidification of casting, the thawing of permafrost, the freezing of foods, aerodynamic heating of missiles and in many other geophysical problems. Mathematically, these problems belong to so-called moving boundary problems in which the moving interface divides the relevant field into at least two regions. Such problems become nonlinear because the location of the moving interface is not known a priori. Due to this nonlinearity analytical solutions can be found only in limited situations, for example, as in Neumann's solution for a one-dimensional problem. Also, the vast majority of theoretical work in this area has been limited to the analysis of one-dimensional moving boundary problems.

To date, several methods are available for the solution of two-dimensional moving boundary problems. In most cases, the emphasis has been placed on a general class of two-dimensional solidification or melting problems, and the following discussion is thus given in the context of this kind of system. Surveys of the early literature with numerous references dating from the time of Stefan are given in Crank's [1] comprehensive book; in Fasano and Primicerio [2] is contained an up-to-date account of mathematical developments and of wide ranging applications to problems in physical and biological sciences, engineering, metallurgy, soil mechanics, decision and control theory, etc. The present note proposes a relatively simple numerical method for the solution of multi-dimensional moving boundary problems on extending and modifying Boadway's [3] transformation. The idea in the present scheme is a particular case of the curvilinear transformation, that is one in which the dependent variable is interchanged with one of the space variables. The variation of this method, the so-called Isotherm Migration Method (IMM) was proposed by Chernousko [4] and independently by Dix and Cizek [5] and subsequently developed and extended to two space dimensions by Crank and co-workers [6–9] and Turland [10]. The use of coordinate transformation for immobilizing the boundary in the case of two-dimensional moving boundary problems has been reported by some of the authors. For example, Furzeland [11] used body-fitted curvilinear coordinate transformation for transforming a curve-shaped region into a fixed rectangular domain; Saitoh [12] and Duda *et al.* [13] discussed several problems using polar coordinates together with the immobilization transformation. More recently, Sparrow and Hsu [14] used coordinate transformations for a control volume formulation. Finally, in their formulation, Gupta and Kumar [15] gave a method based on coordinate transformation which transformed the time varying domain into an invariant one.

In our approach, we propose a relatively simple numerical method for the multi-dimensional moving boundary problems by an independent variable interchange. The present scheme is an extension of Boadway's [3] transformation to time-dependent moving boundary problems in a two or more dimensional case. In our method, not only the shape of the moving interface but also that of the fixed boundary can be selected arbitrarily, thereby allowing its application to more practical situations regardless of the geometry of the problem considered.

2. THE EXTENSION OF BOADWAY'S TRANSFORMATION

For the purpose of illustration, governing equations are presented for the two-dimensional case, since the extension to three dimensions is accomplished in a similar manner. Hence, a particular case of the curvilinear transformation for the heat flow equation can be performed for example, following Boadway's [3] treatment of fluid flow problems. The equation for heat flow in a homogeneous medium in which the heat conductivity k and specific heat c may be functions of temperature U, and density ρ, can be written as

$$c\rho\frac{\partial U}{\partial t}=\frac{\partial}{\partial x}\left(k\frac{\partial U}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial U}{\partial y}\right). \quad (1)$$